Linear Algebra Application in Competitive Gaming for Esports Tournament Seeding

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***Abstract*—The current state of the ever growing esports scene demands a fair system for competition. A part of which is the seeding system for tournaments in which the livelihood of professional teams depends on. This paper aims to explore the use of linear algebra to create a model of a fair and unbiased system to rank teams purely on their performance. Simulated results displayed that the best teams converges at the top, tested against real-world data. This paper concludes that a linear algebraic approach can be used for creating a fair and unbiased seeding system.**

***Index Terms*—Colley matrix, esports, linear algebra, ranking algorithms, systems of linear equations**

I. Introduction

Electronic sports, usually shortened as Esports, is a mode of competition using video games, particularly multiplayer video games. Participants, both individual and teams compete in a series of matches where the loser is disqualified, and the winner proceed to the next match until only a singular participant remains, the winner. League of Legends, a multiplayer video game with a considerable amount of following of its esports scene is an example of how fast esports has grown in terms of popularity [1]

Riot Games, the organizer of the top-level professional league of League of Legends, formed multiple regional leagues; LTA North, LTA South, LEC, LCK, LPL, and LCP. Each of the league contains a variable number of teams, LTA North, LTA South, and LCP have 8, LCK having 9, LEC with 12, and LPL having 15 teams. There is also a global competition where the best teams of each region are selected to represent their own region on the international stage. There are 3 international events, First Stand, MSI, and Worlds. Each international competition is preceded by its own regional qualification called splits.

The format of each split and international stage could be separated into 2 main stages, the group stage and the playoff stage. The group stage is a preliminary process where teams are matched until a set number of teams are remaining, using methods such as round-robin or Swiss-system. After the group stage has ended, each team are ranked and the best previously set number of teams are seeded into a playoff bracket, where each team are going to go head-to-head with other teams until a winner is crowned.

Playoff seeding is a complex process where many factors need to be accounted to ensure a competitively fair result is obtained. A method that has been used historically is using the winning percentage of a team and comparing to other teams and ranking the teams based on the statistics. However, this method of calculating team ranking can be flawed, especially if a stronger team is only playing against teams that are significantly weaker (colloquially known as "seal clubbing"). For example, if team A has a 100% win-rate when playing against only weak teams, whereas team B has 75% win-rate playing against the world champions, team A will be ranked higher than team B. This could result in a statistically unfair ranking.

Linear algebra provides a solution using matrices and linear equations to solve for ratings simultaneously rather than iteratively. Linear systems provide a more statistically fair rankings than traditional win-rate comparison. This paper focuses in the mathematical derivation and implementation of the Colley Matrix with adaptation to fit the League of Legends competition format.

II. Theoretical Basis

A. Systems of Linear Equations

A linear equation can be defined as a combination of n variables $x_1, x_2, x_3, \cdots, x_n$ that can be expressed as

$$a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n = b$$

Where $a_1, a_2, a_3, \cdots, a_n$, and b are constants [2].

A system of linear equations is a group of linear equations, a system with m linear equations and n variables can be represented as

$$\begin{aligned}
a_{11}x_1 + a_{21}x_2 + a_{31}x_3 + \cdots + a_{n1}x_n &= b_1 \\
a_{12}x_1 + a_{22}x_2 + a_{32}x_3 + \cdots + a_{n2}x_n &= b_2 \\
a_{13}x_1 + a_{23}x_2 + a_{33}x_3 + \cdots + a_{n3}x_n &= b_3 \\
&\vdots \\
a_{1m}x_1 + a_{2m}x_2 + a_{3m}x_3 + \cdots + a_{nm}x_n &= b_m
\end{aligned}$$

This structure of linear equations system can also be represented using a matrix equation of $Ax = B$

$$\begin{bmatrix} a_{11} & a_{21} & a_{31} & \cdots & a_{n1} \\ a_{12} & a_{22} & a_{32} & \cdots & a_{n2} \\ a_{13} & a_{23} & a_{33} & \cdots & a_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{1m} & a_{2m} & a_{3m} & \cdots & a_{nm} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_m \end{bmatrix}$$

The resulting equation of matrices where A is the coefficient matrix (the match schedule), x is the unknown vector (the ratings), and B is the constant vector (the match result).

B. Special Matrices

Not all matrix are created equal, some are more special than others. Understanding these special matrices and it's properties are the core how linear algebra is used to solve real-world problems, and in our case, the Colley Matrix.

1) Square Matrix: A matrix with size of $N \times N$ is called a square matrix, since the size of the columns and rows are equal, resembling a square.

$$\begin{bmatrix} 1 & 3 & 5 \\ 1 & 8 & 2 \\ 1 & 4 & 2 \end{bmatrix}$$

2) Identity Matrix: An identity matrix is a special matrix where the diagonal elements of the matrix has a value of one, and any other elements has a value of 0.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3) Symmetric Matrix: A matrix where each element $a_{nm} = a_{mn}$ is called a symmetric matrix. This property results in the matrix equaling itself when transposed ($A = A^T$)

$$\begin{bmatrix} 5 & 7 & 6 & 5 \\ 7 & 10 & 8 & 7 \\ 6 & 8 & 10 & 9 \\ 5 & 7 & 9 & 10 \end{bmatrix}$$

4) Diagonally Dominant Matrix: Diagonally Dominant Matrix has an interesting where if each element of the diagonal is greater than the sum of its row, then the matrix is non-singular, this property can be proven using the Levy-Desplanques Theorem.

1) Assume the matrix A is singular. This implies a solution x such that $Ax = 0$
2) Let x_k be the element in that vector such that ($|x_k| \geq |x_j|$ for all j)
3) For the K-th row of the equation $Ax = 0$ we can find:
$$a_{kk}x_k + \sum_{j \neq k} a_{kj}x_j = 0$$
$$a_{kk}x_k = -\sum_{j \neq k} a_{kj}x_j$$
4) Take the absolute value of both side
$$|a_{kk}||x_k| = \left| \sum_{j \neq k} a_{kj}x_j \right|$$
5) And by the Triangle Inequality, ($\sum |a| \geq |\sum a|$), and since $|x_k|$ is the largest value:
$$|a_{kk}||x_k| \leq \sum_{j \neq k} |a_{kj}||x_j| \leq \sum_{j \neq k} |a_{kj}||x_k|$$
6) Since $|x_k|$ is not zero, we divide both sides:
$$|a_{kk}| \leq \sum_{j \neq k} |a_{kj}|$$
This equation states that the diagonal element is less or equal to the sum of others, and by definition, a Diagonally Dominant matrix requires the inverse, the diagonal element be greater to the sum of other. Therefore, A CANNOT be singular.

C. Determinants and Invertibility

The determinant of a matrix can be defined as a matrix "special number" and relates to the behaviour of the matrix row operations. The determinant matrix with size 2×2 can be calculated as

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
$$det(A) = \big|A\big| = ad - bc$$

For larger matrices, the determinant can be calculated using Laplace expansion. Example for a matrix with size 3×3

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$
$$det(A) = \big|A\big| = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

A matrix with a determinant of 0 is called singular, inversely, if a matrix determinant does not equals 0, then it is a non-singular matrix.

The matrix determinant is integral for matrix inversions. Let A^{-1} be the inverse of A means $AA^{-1} = I$. In other words, the inverse of a matrix is a new matrix such that when multiplied with the original matrix results in a identity matrix.

The inverse of the matrix can be used in our case to calculate the unknown vector x.

$$Ax = b \rightarrow A^{-1}Ax = A^{-1}b \rightarrow Ix = A^{-1}b \rightarrow x = A^{-1}b$$

The matrix equation can be represented as

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{21} & a_{31} & \cdots & a_{n1} \\ a_{12} & a_{22} & a_{32} & \cdots & a_{n2} \\ a_{13} & a_{23} & a_{33} & \cdots & a_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{1m} & a_{2m} & a_{3m} & \cdots & a_{nm} \end{bmatrix}^{-1} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_m \end{bmatrix}$$

D. The Colley Matrix Method

The Colley Matrix method is originally developed by Dr. Wesley Colley for ranking football college teams. The method transforms the problem into a system of linear equation, which can be solved simultaneously ensuring the final rankings are free of path dependence bias.

1) Probability Model and Laplace Rule of Succession: The regular percentage of winning calculation works fine for large data samples, but suffers anomalies when small data samples are used. A team with a record of 1-0 (100%) is ranked higher than a team with a record of 10-1 (90.9%), despite the latter team being a more proven competitor.

This issue can be mitigated using Laplace's Rule of Succession. The rule adds "fictitious" games to each team's record, one win and one loss. The rating r_i for a team i is defined as the probability of winning the next game, not the current frequency of winning.

$$r_i = \frac{1 + w_i}{2 + t_i}$$

Where:

- w_i is the number of wins for team i
- t_i is the number of games team i has played

Applying this rule results in a new team with 0 games played with a neutral 0.5 rating and a team with a 1-0 record a 0.66 rating.

2) The Linear System: The usage of Laplace Rule of Succession solves the issue of small sample sizes, but it does not account for the *strength of schedule*, or simply put it does not care about the strength of the match-up. Integrating the opponent's strength can be done by approximating r_i as the average of the game outcome summed with the opponent's rating. The Colley Matrix can be written as an equation $Cr = b$, with r being the rating vector we need to solve for.

The C being the coefficient matrix with size $N \times N$ and is constructed as follow:

- Diagonal Entries (C_{ii}) represents the total number of games played by team i plus the Rule of Succession factor

$$C_{ii} = 2 + t_i$$

- Off-Diagonal Entries (C_{ij}) represents the interaction between team i and j

$$C_{ij} = -n_{ij}$$

Where n_{ij} is the number of times team i played against team j

The Bias Vector (b) represents the cumulative bias–strength–of teams based on raw win-loss record, adjusted by the Laplace Method.

$$b_i = 1 + \frac{1}{2}(w_i - l_i) \tag{1}$$

Where w_i and l_i is the win and loss record of team i, respectively.

3) Final Rating: The final rating vector r can be obtained by inverting matrix C such that

$$r = C^{-1}b$$

Some properties of C are desirable for a ranking systems:

- Symmetry: since $n_{ij} = n_{ji}$ (Each team plays each other, if team A played team B, then team B also played team A) the matrix is symmetric and hence $C = C^T$ is true.
- Non-Singularity: The matrix C is strictly diagonally dominant because $2 + t_i > \sum n_{ij} = t_i$. As a result, C is always non-singular, and therefore invertible, ensuring that a unique solution exists for any tournament structure.
- Mean Reversion: The ratings of each team are "normalized" and are centered around 0.5, meaning a rating over 0.5 indicates an above average team and vice-versa.

Solving this system results in the rating r_i implicitly capturing the strength of team i's opponents. Defeating a stronger opponent will give team i a higher rating boost compared to defeating a weaker opponent.

III. Implementation

Implementing the Colley Matrix requires a capable data pipeline on acquiring match results, transforming said results into algebraic forms and solving the linear systems. This paper uses the Python programming language due to its extensive ecosystem for scientific computation, data acquisition, and data manipulation.

The implementation is divided into three distinct stages, data acquisition, data processing, and the Colley Matrix computation.

A. Data Acquisition

In order to test the adequacy of the Colley Matrix method for real-world esports environment, match data are collected from gol.gg, a comprehensive database for League of Legends professional tournaments. The collection of matches being used are from all the top-level leagues splits and all international tournaments.

Data extraction is performed using regular Python request library and Pandas a Python library that allows us to parse webpages. The scraping module works by targeting previously targeted DOM (Document Object Model) elements within the tournament list page. The scraper extracts the following properties of each match:

1) Team Names: Unique identifiers for each team playing (e.g., "T1", "Gen.G eSports").
2) Match Result: The outcome of the match for each team (3-1, 0-2).
3) Date: The date that the match took place.

B. Data Processing

After the data acquisition are finished, the raw data are then ingested into Pandas DataFrames for easier processing. This stages also maps specific match results into the mathematical form required for matrix operations.

A list is created to store all the matches, which are gonna be used to construct the Colley Matrix. This step also calculates the winner of the match by comparing the match results between the two teams, the team with the higher score is then classified as the winner.

C. Matrix Computation

The core mathematical operation are handled by NumPy, a highly-optimized library for computing all things mathematical in Python. The construction of the linear system $Cr = b$ follows an algorithmic approach:

1) The $N \times N$ coefficient matrix is initialized with values 2 on its diagonal. The algorithm then iterates over the previously stored matches and decrements the off-diagonal entries by 1 and increments the diagonal entries by 1, corresponding to each team index on the matrix.
2) The bias vector of size N is then created and the algorithm iterate over every matches and increments or decrements each team bias corresponding to their wins and loses, using the formula (1).
3) The system is then solved using NumPy's linear algebra solver (`numpy.linalg.solve`) which returns our rating vector. The vector is then indexed back to the existing list of teams and sorted based on ratings decreasingly.

IV. Simulation and Experiment

In order to truly evaluate the validity and accuracy of the Colley Matrix method for esports, this study conducted two different test, a controlled simulation to test how the Colley Matrix method ranks teams compared to raw win-rate to test the Colley Matrix method ability to remove the strength of schedule bias. The other test is a real-world simulation of the 2025 League of Legends season, this serves the purpose of demonstrating the Colley Matrix method in real-world esports environment.

A. Controlled Simulation

A "tournament" is setup consisting of 4 teams; Cupcake, Titan, Prodigy, and Rookie. The match schedule are designed to expose the flaw in raw win-rate rankings.

1) The four teams are scheduled in specific ways in order to fully test the Colley Matrix method.
   - Team Cupcake will play and win five times against team Rookie in the tournament and only loses once against team Titan and once against team Prodigy. This serves as the team that has a high win-rate but did not really play against any strong opponent, a "seal-clubber" if one might.
   - Team Prodigy is setup to lose against Team Titan and wins against Team Cupcake and Team Rookie. This is the team that hypothetically outrank Team Cupcake in the Colley Matrix method rankings, yet outranked in pure win-rate
   - Team Titan just wins against each other team once.
2) After the simulation has finished, the results are as follow:

Table I: Simulation Result (Win-Rate)

Simulation Result (Win-Rate)			
Team	Record	Win%	Colley Rating
Titan	3-0	100%	.75
Cupcake	5-2	71%	.51
Prodigy	2-1	67%	.58
Rookie	0-7	0%	.15

B. Real-World Simulation

This real-world simulation uses data gathered from the 2025 season of League of Legends esports. In total, there are 967 matches collected across 37 splits and tournaments. The results are as follows:

Table II: Simulation Result (Top 10 Colley Rating)

Simulation Result (Top 10 Colley Rating)			
Team	Record	Win%	Colley Rating
Gen.G eSports	49-8	86%	1.135120
Hanhwa Life eSports	41-18	69%	0.991588
T1	41-19	68%	0.985823
Top Esports	50-17	75%	0.884212
Anyone's Legend	46-19	71%	0.860861
KT Rolster	27-23	54%	0.838090
CTBC Flying Oyster	35-10	78%	0.816425
Bilibili Gaming	44-21	68%	0.814617
Dplus KIA	28-18	61%	0.780510
FlyQuest	30-10	75%	0.771574

Furthermore, a test using only the matches from LCK Round 1-2 Split results are as follow:

Table III: Simulation Result (Top 10 Colley Rating)

Simulation Result (Top 10 Colley Rating)			
Team	Record	Win%	Colley Rating
Gen.G eSports	18-0	100%	0.909091
Hanhwa Life eSports	14-4	78%	0.727273
T1	11-7	61%	0.590909
KT Rolster	11-8	58%	0.566288
Dplus KIA	10-9	53%	0.545455
Nongshim RedForce	10-8	55%	0.524621
BNK FearX	6-12	33%	0.363636
OK BRION	5-13	29%	0.318182
DRX	5-13	29%	0.318182
DN Freecs	1-17	5%	0.136364

V. Result Analysis

A. Controlled Simulation

Comparing the Colley Rating directly with the win-rate, an obvious difference in ranking can be seen. Despite having a higher raw win percentage, Team Cupcake has a lower Colley Rating. This phenomenon directly exposes the flaw of raw win-rate ranking and highlights the Colley Matrix method for bias-free ranking.

B. Real-World Simulation

As for the real-world simulation results, a good comparison of the teams that won an international event in 2025, being Hanhwa Life eSports, Gen.G eSports and T1 are in the top three of the Colley Matrix method rankings. Notable teams with multiple international appearances such as CTBC Flying Oyster, Top Esports, and FlyQuest are also sitting pretty in the top 10.

Comparing the raw win-rate and the Colley Ratings also directly showcases the benefit of the Colley Matrix method ranking benefit of bias-free rating with lower win-rate team KT Rolster ranked 6th based on the Colley Rating. KT was consistent in the mid-season, and simply wakes up and ended up being the runner up for Worlds 2025 behind T1. This ”miracle run” results in KT Rolster gaining a lot of rating before the end of the season, hence the 6th place ranking.

Other consistent teams such as Bilibili Gaming and Dplus KIA are also on the top 10. This points into a strong encouragement teams to focus on keeping a consistent performance and not hard-focusing on win-rate. Matching up against better opponents still means you are getting better.

The results using matches from LCK Round 1-2 are can be used to seed Gen.G eSports, Hanhwa Life eSports, T1 and KT Rolster, Dplus KIA and Nongshim Redforce into the playoffs with the better teams, Gen.G and Hanhwa Life getting an advantage such as an upper bracket start. The fact that Gen.G won MSI after being ranked 1st on the Colley Matrix rating also solidifies the rating system efficacy.

C. Limitations

As most things in the world, this implementation of the Colley Matrix method ranking is not perfect. This model lacks nuance, in the context that competitive gaming has a lot of factor, both in-game; KDA (Kill-Death Ratio), team compositions, game mechanic changes, and out-game; player experience, gaming environment, world events, that this model simply cannot account for.

A proposed change to account for one these factor, KDA, is to change the formula for bias vector into

$$b_i = 1 + \frac{1}{2}((W - L) + \lambda(KDA_i))$$

Where $\lambda(KDA_i)$ is a function that returns a value depending on team i KDA after winning the game.

More changes could be made depending on the game, an example is in Counter-Strike, a first-person shooter game where some teams are bound to be better at some maps. Weighing the off-diagonal entries for a team that is favored to win a specific map so that if they win, they are rewarded less, and penalized more if they lose.

The Colley Matrix method ranking also falls off when massive matchmaking pool is involved. Although the use of sparse matrix can greatly reduce performance drop-off, having a matrix with dimension in the millions is not ideal for computation.

VI. Conclusion

This paper explored linear algebra and its application, specifically the Colley Matrix method, an alternative that has been proven to be superior compared to traditional win-percentage algorithms for ranking competitive esports teams. Modeling a tournament schedule as a linear equation of $Cr = B$, we successfully decoupled a team’s ranking–and its true strength–from a simple percentage and the order they play their matches.

Theoretical analysis verified that the Colley matrix C is strictly diagonally dominant, guaranteeing it as a non-singular matrix and therefore has a unique ranking solution exists for any tournament structure. Using Python-based simulations and applying the method to League of Legends 2025 Season matches data, the system showed that it can reliably identify the strongest teams, even in cases where simple win–loss records gave a distorted picture.

The usage of the Colley Matrix method ranking also can be used for seeding the best teams accordingly. Reflecting on the LCK Round 1-2 Split simulation results, it is important for a competitive environment to make sure that only the best always has the advantage, and the lesser teams must fight to earn that advantage.

From an academic standpoint, this study showcases the elegance of seemingly simple system could result in such interesting real-world applications. This current model for ranking teams not only can be built upon to accommodate more factor, but also to be adapted to other fields, the original Colley Matrix is created to rank football teams afterall. This method for ranking teams are not meant to replace current ranking systems such as the ELO system, but simply to provide an insight that maybe, in order to solve something complex, a simple solution can work.

Other branches of mathematics can be applied to the ranking system that allows many possible combinations of factor, or maybe a more generalized version to rank almost anything. The benefits in understanding how these system works are the key to ensuring a world that works for everyone, fairly.

VII. Appendix

The code for simulating the Colley Matrix method ranking can be found on
https://gist.github.com/PikaProgram/54ffbd33cc295a14a9d14564652e870a

Acknowledgment

The writer expresses his thanks to his family and friends, first and foremost. Mojang for making the Minecraft, GregoriousT and DreamasterXXL for GregTech and GT: New Horizons, respectively. The hit show Arcane for all the emotional rollercoaster it has put me through.

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PERNYATAAN

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Bandung, 24 Desember 2025

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